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G-flux, supersymmetry and Spin(7) manifolds

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Abstract: In this note we study warped compactifications of M-theory on manifolds of Spin(7) holonomy in the presence of background 4-form flux. The explicit expression for the superpotential can be given in terms of the self-dual Cayley calibration on the Spin(7) manifold, in agreement with the general formula proposed in hep-th/9911011.

Keywords: M-Theory, Superstring Vacua.

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Contents

1. Introduction

Various aspects of M-theory compactifications on manifolds of exceptional holonomy and related vacua have been studied recently [1]–[46]. This is partly due to their relation to minimally supersymmetric gauge theories. Although Spin(7) manifolds are perhaps less relevant to the construction of realistic models than G_2 manifolds, they expose other aspects of M-theory, related to the interesting dynamics of the $\mathcal{N} = 1$ effective theory in $2 + 1$ dimensions [4]. In compactification on G_2-manifolds supersymmetry and zero cosmological constant require the 4-form field strength G to vanish [48, 49, 50], whereas — as we will see presently — in M-theory on Spin(7)-manifolds there is more freedom, and non-trivial G-flux can be consistent with supersymmetry.

In fact, there are several reasons why non-trivial G-flux may be required in M-theory compactifications on Spin(7)-manifolds. For example, cancellation of membrane anomalies in an arbitrary vacuum spacetime forces the 4-form flux G to obey the modified quantization condition [47]:

$$\left[\frac{G}{2\pi}\right] - \frac{\lambda}{2} \in H^4(X, \mathbb{Z})\,, \tag{1.1}$$

where the integral class $\lambda = p_1(X)/2 \in H^4(X; \mathbb{Z})$ and X is the compactification manifold. If λ is even, then $G = 0$ is a consistent part of the vacuum data. In particular, in M-theory on G_2 holonomy manifolds, if the anomaly did not vanish the corresponding compactifications would not lead to supersymmetric vacua because G would have to be non-zero. On the other hand, if $\dim(X) \geq 8$, then the above anomaly may not vanish, in which case one has to turn on background G-flux. This typically happens in M-theory on 8-manifolds of Spin(7) holonomy and leads to interesting physics [4].

Another, closely related condition that in general requires the G-flux to be non-zero in compactification on an 8-manifold X is the global tadpole anomaly [51]:

$$\frac{\chi(X)}{24} = N_{M2} + \frac{1}{2}\int_X \frac{G\wedge G}{(2\pi)^2}\,. \tag{1.2}$$

Here, $\chi(X)$ is the Euler number of X and N_{M2} is the number of space-filling membranes. Clearly, this anomaly is trivialised if the dimension of X is less than eight. We conclude that, in general, background G-flux is required in compactifications of M-theory on manifolds X of dimension 8 (or greater). For instance, if we consider vacua without membranes and 8-manifolds with non-zero Euler number then non-zero G-flux is required. Therefore, it is important to study which such compactifications can be supersymmetric and, if so, what the corresponding supersymmetry conditions are.

In this note we consider manifolds X with metric g_X whose holonomy group is Spin(7) (or a subgroup theorof). We obtain an $\mathcal{N}=1$ supersymmetric theory when $\mathrm{Hol}(g_X) = \mathrm{Spin}(7)$. We find that the background flux generates an effective superpotential of the following simple form, originally proposed in [52]:

$$W = \int_X G\wedge\Omega\,. \tag{1.3}$$

Here Ω is the self-dual closed Spin(7)-invariant 4-form which exists on any manifold of Spin(7)-holonomy.

Assuming that the typical size of X is much larger than the Planck length l_{Pl}, in the rest of this letter we show a complete agreement between supersymmetry conditions in the eleven-dimensional supergravity and in the effective three-dimensional theory with superpotential W. We find that only particular choices of G-flux — characterised by a particular representation of Spin(7) are allowed — if we require that the resulting theory does not break supersymmetry spontaneously to leading order. The conditions for unbroken supersymmetry in supergravity have previously been studied in [55, 56].

In addition to the potential for scalar fields, we show that the abelian gauge fields in three dimensions which originate from the C-field in eleven dimensions can gain a mass due to G-flux induced Chern-Simons couplings. We also briefly comment on corrections to the leading potential including those due to membrane and fivebrane instantons.

To conclude the introduction we will describe some elementary aspects of the cohomology of Spin(7) manifolds which we will require in our analysis of supersymmetric vacua. For more details on the geometry of special holonomy manifolds we recommend [53].

1.1 Cohomology of Spin(7) manifolds

On a riemannian manifold X, whose metric g has holonomy H, all fields (i.e. vectors, p-forms, spinors) on X form representations of H. With particular regard to p-forms on X this decomposition of forms commutes with the laplacian and hence the cohomology groups of X are arranged into representations of H. For example, the Hodge-Dolbeaut cohomology groups $H^{p,q}(X,\mathbb{R})$ of a Kahler manifold X consist of harmonic forms on X in a particular representation of $H = U(d/2)$.

For X a manifold of Spin(7)-holonomy we obtain the following decompositions of $H^k(X,\mathbb{R})$ which are induced from the decomposition of $\Lambda^k(\mathbb{R}^8)$ into irreducible representations of Spin(7):

$$\begin{aligned}
H^0(X,\mathbb{R}) &= \mathbb{R}\\
H^1(X,\mathbb{R}) &= H^1_{\mathbf{8}}(X,\mathbb{R})\\
H^2(X,\mathbb{R}) &= H^2_{\mathbf{7}}(X,\mathbb{R}) \oplus H^2_{\mathbf{21}}(X,\mathbb{R})\\
H^3(X,\mathbb{R}) &= H^3_{\mathbf{8}}(X,\mathbb{R}) \oplus H^3_{\mathbf{48}}(X,\mathbb{R})\\
H^4(X,\mathbb{R}) &= H^4_{\mathbf{1}^+}(X,\mathbb{R}) \oplus H^4_{\mathbf{7}^+}(X,\mathbb{R}) \oplus H^4_{\mathbf{27}^+}(X,\mathbb{R}) \oplus H^4_{\mathbf{35}^-}(X,\mathbb{R})\\
H^5(X,\mathbb{R}) &= H^5_{\mathbf{8}}(X,\mathbb{R}) \oplus H^5_{\mathbf{48}}(X,\mathbb{R})\\
H^6(X,\mathbb{R}) &= H^6_{\mathbf{7}}(X,\mathbb{R}) \oplus H^6_{\mathbf{21}}(X,\mathbb{R})\\
H^7(X,\mathbb{R}) &= H^7_{\mathbf{8}}(X,\mathbb{R})\\
H^8(X,\mathbb{R}) &= \mathbb{R}\,.
\end{aligned} \tag{1.4}$$

The additional label "$\pm$" denotes self-dual/anti-self-dual four-forms, respectively. The cohomology class of the 4-form Ω generates $H^4_{\mathbf{1}^+}(X,\mathbb{R})$. We will denote the dimension of $H^k_{\mathbf{r}}(X,\mathbb{R})$ as $b^k_{\mathbf{r}}$.

Thus far, we have only used the Spin(7)-structure locally. The fact that the metric on X has Spin(7)-holonomy implies global constraints on X and this forces some of the above groups to vanish when X is compact. It will prove crucial to determine which ones.

The reason we are interested in Spin(7)-manifolds in M-theory is that they admit one covariantly constant (or parallel) spinor. This is the condition for minimal supersymmetry in three dimensions in the absence of G-flux. Since the metric on X has Spin(7) holonomy, it is Ricci flat and so

$$D^2 = \nabla^2 \tag{1.5}$$

where D is the Dirac operator and ∇ the covariant derivative. Therefore, a zero mode of the Dirac operator is necessarily a constant spinor and vice-versa. Thus, we learn that the kernel of the Dirac operator on a manifold of Spin(7) holonomy is one dimensional.[1] In fact, the index of the Dirac operator on a manifold with exactly Spin(7) holonomy is precisely one [53]. This follows from the equation above and the fact that manifolds of Spin(7) holonomy have a constant spinor of only one chirality. Therefore, on a manifold of Spin(7) holonomy the cokernel of the Dirac operator is empty. Now, we will use the fact that spinors of any chirality on a manifold of Spin(7)-holonomy can actually be identified with certain combinations of p-forms — a fact which follows essentially from $\mathbf{8_s} \to \mathbf{1} + \mathbf{7}$ and $\mathbf{8_c} \to \mathbf{8}$ when SO(8) $\to$ Spin(7).

Namely, if $S = S_+ \oplus S_-$ is a spin bundle on X, we have a natural isomorphism [53]:

$$S_+ \cong \Lambda^0_{\mathbf{1}} \oplus \Lambda^2_{\mathbf{7}}\,, \qquad S_- \cong \Lambda^1_{\mathbf{8}}\,. \tag{1.6}$$

[1] Of course, on manifolds such as Calabi-Yau fourfolds which are Spin(7) manifolds whose holonomy is a proper subgroup of Spin(7) there are more zero modes.

Furthermore, one can identify the Dirac operator $D: C^\infty(S_+) \to C^\infty(S_-)$ with the following operator acting on differential forms:

$$\pi_8 \circ d: C^\infty(\Lambda^0_{\mathbf{1}} \oplus \Lambda^2_{\mathbf{7}}) \to C^\infty(\Lambda^1_{\mathbf{8}}) \tag{1.7}$$

i.e. we take the exterior derivative and project the result onto the eight-dimensional representation of Spin(7).

Therefore, the Dirac index (also called the A-roof genus) on the compact Spin(7) manifold X can be written:

$$\hat{A}(X) = b^0_{\mathbf{1}} + b^2_{\mathbf{7}} - b^1_{\mathbf{8}} = 1 + b^2_{\mathbf{7}} - b^1_{\mathbf{8}}\,. \tag{1.8}$$

In particular, if $\mathrm{Hol}(g_X) = \mathrm{Spin}(7)$, X is simply-connected and as we saw above has $\hat{A}(X) = 1$. Therefore, we have $b^1_{\mathbf{8}} = 0$ and $b^2_{\mathbf{7}} = 0$. Using the canonical isomorphisms (which are easily obtained by wedging and contracting with Ω) [53]:

$$\Lambda^1_{\mathbf{8}} \cong \Lambda^3_{\mathbf{8}} \cong \Lambda^5_{\mathbf{8}} \cong \Lambda^7_{\mathbf{8}}\,, \qquad \Lambda^2_{\mathbf{7}} \cong \Lambda^4_{\mathbf{7}} \cong \Lambda^6_{\mathbf{7}} \tag{1.9}$$

we obtain further constraints $b^1_{\mathbf{8}} = b^3_{\mathbf{8}} = b^5_{\mathbf{8}} = b^7_{\mathbf{8}} = 0$ and $b^2_{\mathbf{7}} = b^4_{\mathbf{7}} = b^6_{\mathbf{7}} = 0$.

To summarize, if X is a compact 8-manifold, such that $\mathrm{Hol}(g_X) = \mathrm{Spin}(7)$, then the cohomology of X can be decomposed into the following representations of Spin(7):

$$\begin{aligned}
H^0(X,\mathbb{R}) &= \mathbb{R} \\
H^1(X,\mathbb{R}) &= 0 \\
H^2(X,\mathbb{R}) &= H^2_{\mathbf{21}}(X,\mathbb{R}) \\
H^3(X,\mathbb{R}) &= H^3_{\mathbf{48}}(X,\mathbb{R}) \\
H^4(X,\mathbb{R}) &= H^4_{\mathbf{1}^+}(X,\mathbb{R}) \oplus H^4_{\mathbf{27}^+}(X,\mathbb{R}) \oplus H^4_{\mathbf{35}^-}(X,\mathbb{R}) \\
H^5(X,\mathbb{R}) &= H^5_{\mathbf{48}}(X,\mathbb{R}) \\
H^6(X,\mathbb{R}) &= H^6_{\mathbf{21}}(X,\mathbb{R}) \\
H^7(X,\mathbb{R}) &= 0 \\
H^8(X,\mathbb{R}) &= \mathbb{R}\,.
\end{aligned} \tag{1.10}$$

In this list, the largest representation structure appears in degree 4. Since we are going to consider M-theory backgrounds with non-trivial 4-form flux G, this cohomology group also plays an important role in our discussion. In particular, it will be crucial that on a compact manifold X of exactly Spin(7) holonomy we have $H^4_{\mathbf{7}^+}(X,\mathbb{R}) = 0$.

2. Supersymmetry conditions in $D = 11$ supergravity

Now we consider the conditions for unbroken supersymmetry in eleven-dimensional supergravity on a Spin(7) manifold X. The supergravity approximation to M-theory is valid as long as the size of X is large, compared to the Planck scale. Supersymmetry conditions in (warped) compactifications of M-theory to three dimensional Minkowski space-time have

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already been discussed in the literature [54] and in fact the conditions for unbroken supersymmetry upon compactification on a Spin(7) manifold have also been obtained [55, 56, 57]. Also, the solutions to the equations of motion of M-theory on Kahler 8-manifolds have been discussed in [58]. In this section, we slightly extend the analysis of supersymmetric vacua, allowing for the possibility that the three-dimensional cosmological constant is non-zero. In other words, we assume the eleven-dimensional space-time to be of the form:

$$M^3 \times X \,,$$

where M^3 is a maximally symmetric three-dimensional space. More precisely, we consider a warped product of M^3 and X, rather than a direct product. If we denote the scalar warp factor $\Delta(y^m)$, then the corresponding metric reads:

$$ds^2 = e^{2\Delta/3}\eta_{\mu\nu}(M^3)\ dx^\mu dx^\nu + e^{-\Delta/3} g_{mn}(X)\ dy^m dy^n \,. \tag{2.1}$$

The external components of the 3-form field C have the form:

$$C_{012} = -e^{\Delta} \,. \tag{2.2}$$

Finally, we put no restrictions on the internal components of the G-flux.

Since all fermionic fields vanish in the background, we can focus only on the supersymmetry variation of the gravitino field:

$$\delta\psi_M = \nabla_M \eta - \frac{1}{288} G_{PQRS}(\Gamma_M^{\ PQRS} - 8\delta_M^P \Gamma^{QRS})\eta \,, \tag{2.3}$$

where η is a supersymmetry variation parameter.

Now we require $\delta\psi_M = 0$ and consider different components of this equation. Since the calculation is pretty standard (see e.g. [54]), here we only outline the main steps. First, one makes the $3+8$ split, compatible with the metric (2.1):

$$\Gamma_\mu = e^{\Delta/3}(\gamma_\mu \otimes \gamma_9) \,, \qquad \Gamma_m = e^{-\Delta/6}(\mathbf{1} \otimes \gamma_m) \,. \tag{2.4}$$

Similarly, one can decompose the supersymmetry parameter η into an eight-dimensional spinor ξ on X (such that $\xi^T\xi = 1$ and $\gamma_9\xi = +\xi$) and into a three-dimensional spinor ϵ on M^3, which obeys[2] $\nabla_\mu \epsilon = m_\psi \gamma_\mu \epsilon$. Specifically, we have:

$$\eta = e^{\Delta/6}(\epsilon \otimes \xi) \,. \tag{2.5}$$

After rescaling transformations that eliminate the dependence on the warp factor Δ, from the internal components of the supersymmetry variation (2.3) we obtain the following supersymmetry condition:

$$m_\psi \gamma_m \xi - \frac{1}{12} G_{mpqr}\gamma^{pqr}\xi = 0 \,. \tag{2.6}$$

[2]In the three-dimensional effective supergravity theory, m_ψ has interpretation as a gravitino mass parameter, which is related to the cosmological constant in the usual way.

However, it turns out to be compatible with the external components of (2.3) if and only if $m_\psi = 0$, i.e. when three-dimensional space-time is flat. Hence, the supersymmetry conditions take the form obtained earlier in [54]:

$$G_{mpqr}\gamma^{pqr}\xi = 0\,.$$

If we multiply this relation by γ^n and by ξ^T from the left and use the identity:

$$\Omega_{mnpq} = \xi^T\gamma_{mnpq}\xi \tag{2.7}$$

we can express this supersymmetry condition in terms of the Cayley 4-form Ω:

$$G_{mpqr}\Omega_n{}^{pqr} = 0\,. \tag{2.8}$$

It is convenient to denote the left-hand side of this equation as $T_{mn} = G_{mpqr}\Omega_n{}^{pqr}$. Then, the above supersymmetry condition reads:

$$T_{mn} = 0\,. \tag{2.9}$$

Let us analyze different components of these equations. T_{mn} is a 2-index tensor field on X. Since g_X has Spin(7) holonomy, we can consider decomposing T into irreducible Spin(7) representations. Which representations appear? If g_X had generic, i.e. SO(8) holonomy, then T decomposes into traceless symmetric, antisymmetric, and trace components. As SO(8) representations these have dimensions $\mathbf{35}, \mathbf{28}$ and $\mathbf{1}$ respectively. But as Spin(7) representations, the $\mathbf{35}$ and $\mathbf{1}$ remain irreducible, whilst $\mathbf{28}$ becomes $\mathbf{7}+\mathbf{21}$.

Now, T is not an arbitrary 2-tensor, but a tensor constructed from two 4-forms G and Ω. The fact that Ω is in the trivial representation of Spin(7) implies that the representations in which T resides can at most be those of G. Then, the fact that 4-forms on a Spin(7) manifold can only be in the representations $\mathbf{1}$, $\mathbf{7}$, $\mathbf{27}$ or $\mathbf{35}$ implies that the antisymmetric part of T cannot contain any component in the representation $\mathbf{21}$. Therefore, we learn that the condition $\mathrm{tr}\,T = 0$ means that G is not a Spin(7) singlet. In other words $G_{\mathbf{1}^+} = 0$. The condition that the symmetric part of T_{mn} vanishes implies that G is self-dual, i.e. the $\mathbf{35}$ piece of G must vanish. Finally, the condition that the antisymmetric part of T_{mn} vanishes says that the $\mathbf{7}$ piece of G vanishes. Therefore, according to (1.10), the 4-form field G compatible with $\mathcal{N}=1$ supersymmetry can have non-vanishing components only in the $\mathbf{27}$ representation of Spin(7):

$$G \in H^4_{\mathbf{27}^+}(X,\mathbb{R})\,. \tag{2.10}$$

Before we proceed to the interpretation of this supersymmetry condition in the effective $\mathcal{N}=1$ three-dimensional theory, let us remark that since it is derived as a local condition on the G-field the result is valid even when X is a non-compact Spin(7) manifold. Such manifolds usually appear as local models in the study of Spin(7) singularities [4] and play an important role in the geometric engineering of $\mathcal{N}=1$ three-dimensional gauge theories decoupled from gravity.

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3. Interpretation in the effective three-dimensional theory

In this section we will interpret the above results in terms of the effective $\mathcal{N}=1$ three-dimensional theory. What is the effective three dimensional theory? When X is large, and G is zero, standard Kaluza-Klein analysis applies and it is straightforward to see [59] that the three dimensional low energy theory is $\mathcal{N}=1$ supergravity with $b^2_{\mathbf{21}}$ vector multiplets A_i (whose vectors arise from the 3-form potential C). There are also $b^3_{\mathbf{48}}$ scalar multiplets ρ_j from the 3-form and $b^4_{\mathbf{35}}+1$ scalar multiplets ϕ_k from the metric tensor. The latter fields parametrise locally the space of Spin(7) holonomy metrics on X which are near g_X. The gauge group is locally $\mathrm{U}(1)^{b^2_{\mathbf{21}}}$ but globally $H^2(X,\mathrm{U}(1))$. As we discussed in the introdution, however, the theory without G-flux may not be a consistent M-theory vacuum. We can regard the theory with G-flux as adding extra couplings to the above theory in which the A,ρ and ϕ fields are massless and non-interacting to leading order in l_{pl}.

The small amount of supersymmetry allows for a rich dynamical structure in these theories, and a variety of interaction terms in the effective lagrangian. For this reason, it is convenient to write the effective lagrangian in superspace, which makes $\mathcal{N}=1$ supersymmetry manifest. Minimal three-dimensional superspace can be obtained[3] by combining three-dimensional coordinates x^μ with real Grassmann variables θ_α, and by introducing the corresponding covariant derivatives D_α. Then, the effective three-dimensional lagrangian can be schematically written as a full superspace intagral:

$$L_{3D}=\int d^3x d^2\theta E^{-1}K+\int d^3x d^2\theta E^{-1}W(\rho_j,\phi_k)\,. \tag{3.1}$$

where the first term represents the kinetic action, while W (unlike K) depends only on the scalar fields but not their derivatives. After we perform $d^2\theta$ integral in (3.1), the last term leads to the scalar potential in the effective theory [60].

In a supersymmetric vacuum with zero cosmological constant, the following conditions must be satisfied:

$$W=0\,,\qquad \frac{\partial W}{\partial\rho_j}=\frac{\partial W}{\partial\phi_k}=0\,. \tag{3.2}$$

These are the supersymmetry conditions that we want to compare to the ones in eleven-dimensional supergravity.

Following [52, 50], we interpret the supersymmetry condition (2.10) in terms of the effective superpotential W induced by the G-flux:

$$W=\frac{1}{2\pi}\int_X G\wedge\Phi\,. \tag{3.3}$$

In general, the expression (1.3) for the effective superpotential was conjectured from the identification of BPS domain walls with branes wrapped on supersymmetric submanifold $S\subset X$. In our case, these are M5-branes wrapped on Cayley 4-cycles, with tension:

$$T\geq\int_S\Phi=|\Delta W|\,. \tag{3.4}$$

[3] [60] is a useful reference.

Here, we will justify the formula (3.3) for the effective superpotential by showing that the eleven-dimensional supersymmetry conditions (2.10) and the corresponding conditions (3.2) in the effective three-dimensional theory are the same.

The first equation in (3.2), namely $W = 0$, implies that three-dimensional cosmological constant is zero, and from (3.3) (which is proportional to $\mathrm{tr}\, T_{mn}$ from the previous section) we find that it requires the singlet piece of G to vanish.

On the other hand, $\partial W / \partial \phi_i$ is the variation of W with respect to the scalar fields which come from the metric deformations of the compact Spin(7) holonomy manifold X. The superpotential W can only depend on these scalars since it only depends on Ω. According to [53], the latter generate $H^4_{\mathbf{35}^-}(X, \mathbb{R})$,[4] so that the second equation in (3.2) implies $G_{\mathbf{35}^-} = 0$. Since for a compact Spin(7) manifold $H^4_{\mathbf{7}^+}(X, \mathbb{R}) = 0$, we conclude that G-flux has to be an element of $H^4_{\mathbf{27}^+}(X, \mathbb{R})$, in complete agreement with the supergravity result (2.10):

$$G \in H^4_{\mathbf{27}^+}(X, \mathbb{R})\,. \tag{3.5}$$

3.1 Quantum corrections to the potential

The expression (1.3) represents the leading contribution to the potential induced by the non-trivial G-flux. In this section we will briefly discuss the "perturbative" and non-perturbative contributions to the potential.

The total superpotential schematically can be written as:

$$W_{\mathrm{tot}} = W + W_{\mathrm{pert}} + W_{\mathrm{non-pert}}\,, \tag{3.6}$$

where W is the classical term (3.3). We will first discuss the perturbative contributions and then the non-perturbative ones. The one-loop contribution to the perturbative superpotential W_{pert} is expected to be in the following simple form:

$$W_{\mathrm{pert}} = \frac{1}{4\pi} G^{ab} \frac{\partial^2 W(\phi)}{\partial \phi^a \partial \phi^b} = \frac{1}{8\pi^2} \int_X G \wedge \delta^2 \Phi + \cdots\,, \tag{3.7}$$

where G^{ab} is a scalar field metric. This follows essentially from the supersymmetry of the theory. The best way to demonstrate this is to compactify the three-dimensional theory further on a circle. This leads to a supersymmetric field theory in two dimensions, which can be also thought of as a result of type-IIA compactification on X. The 'classical' superpotential in this theory also has the form (3.3), whereas (3.7) is a one-loop anomaly [61]. In fact, [61] argue that there are no additional contributions at higher loop order.

The non-perturbative part of the superpotential is generated by $M2$-brane and $M5$-brane instantons wrapped on three and six-cycles, V^3 and V^6, respectively. Since such cycles are not supersymmetric, such instantons break both supersymmetries. Consequently, the two corresponding Goldstone fermions imply that these wrapped branes will contribute to W. The form of these contributions is of the form

$$W_{\mathrm{non-pert}} \sim \sum_{V^3} e^{-\mathrm{Vol}(V^3) - \int_{V^3} C} + \sum_{V^6} e^{-\mathrm{Vol}(V^6) - \int_{V^6} \hat{C}}\,. \tag{3.8}$$

[4] In other words, if we add any small harmonic anti-self-dual 4-form to Ω we get a new Spin(7) structure and a correspondingly new Spin(7) holonomy metric.

Here C is the three-form field and $\hat{C}$ is its dual. Note that the period of C through V^3 is a function of the scalars ρ_j and that the period of $\hat{C}$ is formally a function of the scalars which are dual to the photon fields. It is conceivable that for generic G-flux the total superpotential $W_{\rm tot}$ in the $\mathcal{N}=1$ effective three-dimensional theory has only isolated fixed points.

Finally, we remark that the $\mathrm{U}(1)^{b^2}$ gauge fields are also typically massive in the presence of G-flux due to Chern-Simons couplings [4]. Inserting the Kaluza-Klein ansatz for the C-field,

$$C = \Sigma_I \alpha_I \wedge A^I(x) + \cdots \tag{3.9}$$

into the interaction,

$$\int C \wedge G \wedge G \tag{3.10}$$

gives rise to the following three-dimensional Chern-Simons action for the three-dimensional gauge fields A^I

$$C_{IJ} \int A^I \wedge dA^J \,, \tag{3.11}$$

where

$$C_{IJ} = \int_X \alpha_I \wedge \alpha_J \wedge G \,, \tag{3.12}$$

where G is the background G-flux. For generic enough G the couplings C_{IJ} will be non-zero and therefore all the gauge fields gain a mass. For instance, if we take $G = \Omega$ then all the diagonal couplings C_{II} are non-zero and are in fact given by

$$C_{II} = -2 \int_X \alpha_I \wedge * \alpha_I \,. \tag{3.13}$$

This follows from the fact that the 2-forms α_I are all in the **21**-dimensional representation of Spin(7) and as such they satisfy

$$-\alpha_I \wedge \Omega = 2 * \alpha_I \,. \tag{3.14}$$

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